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Quantum aspects of gauge theories, supersymmetry and unification.

Supergravity Predictions on Conformal Field Theories*

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Abstract: We give an update on recent results about the matching between CFT operators and KK states in the AdS/CFT correspondence, and add some new comments on the realization of the baryonic symmetries from the supergravity point of view.

Keywords: Supergravity, Superstrings, CFT.

1. Introduction

Over two years have passed since the proposal of Maldacena [1] of a correspondence between supergravity and string theories on Anti de Sitter (AdS) spaces and conformal field theories (CFT) on their boundary, in large N limits. During this period, this conjectured relation has been expressed more precisely [2, 3], it has been investigated under many aspects, partially verified in various cases and also extended in different directions [4].

One of the tests which has been carried out in great depth, giving also some unexpected new results, is the matching of the spectra of conformal operators on the CFT side with the Kaluza–Klein (KK) excitations in the compactified supergravity. The AdS/CFT correspondence indeed predicts a fixed relation between scaling dimensions and KK mass modes, which can be tested in many examples. This matching was first proposed and used in [3], where it was called the "comparison to experiment" of the AdS/CFT conjecture. In a first stage, it had been performed only for the maximal supersymmetric cases (i.e. compactifications on spheres) and for the lower supersymmetric models deriving from orbifold compactifications [4].

Our group has extensively focused on the generalization of the spectra matching test to lower supersymmetric models obtained by supergravity compactifications on the product of AdS space with various Einstein manifolds [5, 6, 7]. Due to the presence of extra global symmetries inherited from the isometries of the internal manifold, beside the R-symmetries, these models have a far richer structure and thus yield much more probing proofs of the AdS/CFT conjecture. In spite of the greater technical complexity of lower (super)symmetric cases, we have chosen to engage in their thorough study because we had at our disposal quite powerful tools for supergravity analysis, such as harmonic expansion on coset manifolds, that were developed in the old days in the context of KK reduction of supergravity models [8]. We would like to collect here our main results and provide a brief resumé of the lessons we have learned by exploring this subject.

2. A test of the correspondence

In the investigation of supergravity theories with lower supersymmetry given by compactifications on coset manifolds, one encounters a very interesting and elaborate multiplet structure which makes possible some non–trivial checks in the

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correspondence with the spectra of conformal operators of the boundary field theory. In fact, differently from the spheres, where all KK modes belong to short representations of supersymmetry and thus have mass values that are protected against quantum corrections, for less symmetric cosets one also finds long and semilong representations, that in principle do not have any protection mechanism to prevent them from running with the couplings. It is thus quite remarkable that one can nevertheless establish a full map between each kind of KK multiplet and appropriate families of conformal operators and their descendants.

We have essentially explored two directions: the correspondence AdS_5/CFT_4 and AdS_4/CFT_3.

The AdS_5/CFT_4 case is more directly relevant from a physical point of view, since it involves four dimensional gauge theories, but also the AdS_4/CFT_3 one offers some intriguing challenges which could give us more insight in the formulation of the conjecture.

For spontaneous compactifications of type IIB supergravity on a five dimensional coset manifold, there is only one space preserving some supersymmetry[9]:

$$T^{11} \equiv \frac{SU(2) \times SU(2)}{U(1)}$$

where the $U(1)$ factor is embedded diagonally in the two $SU(2)$. We have determined the full KK spectrum on this manifold [10] (extending some previous partial results [11, 12]) and then tested the AdS/CFT map [5], by matching it against the spectrum of primary conformal operators of the dual CFT constructed in [13]. In this example we have not only shown that the duality works, but we have also some new hints on the CFT behaviour.

The extension of such study to M–theory compactifications on seven–manifolds is much more complicated. It is indeed known that three dimensional CFT's are difficult to analyze because they emerge in non–perturbative limits of conventional gauge field theories. Moreover, if for type IIB on T^{11} we had a well defined CFT to be used towards the comparison, for the M–theory compactifications a well established CFT was not available. Thus we have *used* the correspondence, at first to guess these CFT's, and then to verify by matching the spectra whether they were well–defined.

M–theory allows a variety of supersymmetric compactifications down to four dimensions. The $\mathcal{N} = 2$ examples can be divided into two categories [14]: toric ones

$$M^{111} \equiv \frac{SU(3) \times SU(2) \times U(1)}{SU(2) \times U(1) \times U(1)} \quad \text{and}$$

$$Q^{111} \equiv \frac{SU(2) \times SU(2) \times SU(2)}{U(1) \times U(1)}$$

and non–toric ones

$$V_{(5,2)} \equiv \frac{SO(5)}{SO(3)}.$$

While for the first, toric geometry helps in the definition of the CFT [6], in the non–toric case one has to deal with even harder difficulties [15].

Summing up, in this analysis we have met three main features which are worth describing in some detail: i) the agreement between the CFT expectations and the supergravity results, ii) the existence of long multiplets with rational energy quantum numbers predicted by supergravity, and iii) the identification of the baryonic symmetries as those deriving from the well known presence of Betti multiplets [16] in the compactified supergravity.

3. Matching the spectra

The AdS/CFT correspondence can be used in two ways: either to control the validity of the CFT by predicting properties of the supergravity, such as the mass spectrum, or to obtain information from tree level calculations in supergravity on the strong coupling CFT behaviour.

Not only the fixed relation required by the AdS/CFT map between the anomalous dimension of the various boundary conformal fields and the masses of the bulk KK modes holds for lower supersymmetry as for the highest symmetric cases [5, 6, 15], but there exists a full correspondence between *all* the KK modes and the conformal operators of preserved scaling dimension.

In order to give a taste of how this works, we turn to the simplest non–trivial example, that is type IIB compactification on $AdS_5 \times T^{11}$.

The dual four–dimensional CFT was given in [13] as an $\mathcal{N}=1$ Yang–Mills theory with a flavor symmetry $G = SU(2)\times SU(2)$. It should describe the physics of a large number (N) of $D3$–branes placed at the singular point of the cone over the T^{11} manifold in the decoupling limit.

The "singleton" degrees of freedom of the CFT, called A and B, are each a doublet of the G factor groups and have a conformal anomalous dimension $\Delta_{A,B} = 3/4$. The gauge group $\mathcal{G}$ is $SU(N)\times SU(N)$ and the A and B chiral multiplets transform in the $(N,\overline{N})$ and $(\overline{N},N)$ of $\mathcal{G}$ respectively. The gauge potentials lie in the adjoint of one of the two $SU(N)$ groups, and their field–strength in superfield notation is given by W_α. They are singlet of the matter groups, with R–symmetry charge $r=1$ and $\Delta = 3/2$.

There is also a superpotential given by [13]

$$\mathcal{W} = \lambda\epsilon^{ij}\epsilon^{kl}\, Tr(A_iB_kA_jB_l), \qquad (3.1)$$

which has $\Delta = 3$, $r=2$. It plays an important role in the discussion in that it determines to some extent both the chiral spectrum and the marginal deformations of the SCFT.

Knowing the fundamental degrees of freedom of the conformal field theory, one could try to write the conformal operators by simply combining the above fields into all possible products while respecting the symmetries of the theory.

The first operators one can build in this way are the chiral operators

$$Tr(AB)^k \qquad (3.2)$$

which are those with the lowest possible dimension for a given R–charge (they have indeed $\Delta = \frac{3}{2}r = \frac{3}{2}k$). We notice that in the (3.2) operators we can freely permute all the A's and B's by using the equations for a critical point of the superpotential

$$B_1\ A_k\ B_2 = B_2\ A_k\ B_1\ ,\, A_1\ B_l\ A_2 = A_2\ B_l\ A_1\ . \qquad (3.3)$$

Next to these, one could also have an operator given by $Tr[W_\alpha(AB)^k]$ or $Tr[W^2(AB)^k]$, and so on. But which are the operators with protected dimension? This is a crucial question, since only the protected operators find a matching state among the KK fields, while those that suffer from quantum corrections are to be found within the full string theory.

It is a well–established result that operators with protected conformal dimension correspond to the short representations of the supergroup which they belong to.

For T^{11}, this supergroup is $SU(2,2|1)$, while for the previously mentioned M–theory cases it is $OSp(4|2)$. More generally, for N supersymmetries the four dimensional case involves $SU(2,2|\mathcal{N})$ whose shortening conditions in terms of superfields have been explained in [17], while the generic three–dimensional case involves $OSp(4|\mathcal{N})$ whose shortenings have been recently discussed in [18].

In the T^{11} example (α, $\dot\alpha$ are spinor indices. x,θ and $\bar\theta$ are the bosonic and fermionic coordinates) we have only three types of such operators, namely the *chiral*

$$\bar{D}^{\dot\alpha} S_{\alpha_1\ldots\alpha_{2J}} = 0, \qquad (3.4)$$

conserved

$$\begin{aligned} \bar{D}^{\dot\alpha_1} J_{\alpha_1\ldots\alpha_{2J_1},\dot\alpha_1\ldots\dot\alpha_{2J_2}} &= 0 \qquad (3.5)\\ \text{and } D^{\alpha_1} J_{\alpha_1\ldots\alpha_{2J_1},\dot\alpha_1\ldots\dot\alpha_{2J_2}} &= 0 \qquad (3.6)\end{aligned}$$

and *semi–conserved* superfields

$$\begin{aligned} &\bar{D}^{\dot\alpha_1} L_{\alpha_1\ldots\alpha_{2J_1},\dot\alpha_1\ldots\dot\alpha_{2J_2}}(x,\theta,\bar\theta) = 0, \qquad (3.7)\\ &(\bar{D}^2 L_{\alpha_1\ldots\alpha_{2J_1}} = 0 \text{ for } J_2 = 0).\end{aligned}$$

These differential constraints imply that these fields satisfy certain specific restrictions on their quantum numbers. As a consequence, their anomalous dimension is fixed in terms of their spin and R–symmetry charge. These constraints are respectively:

$$r = \frac{2}{3}\Delta, \qquad (3.8)$$

for chiral ones,

$$r = \frac{2}{3}(\Delta - 2 - 2J_2) \qquad (3.9)$$

for semiconserved ones and

$$\begin{aligned} r &= \frac{2}{3}(J_1 - J_2),\\ \Delta &= 2 + J_1 + J_2,\end{aligned} \qquad (3.10)$$

for conserved ones.

It is easy to relate operators of different type by superfield multiplication. The product of a

chiral $(J_1, 0)$ and an anti–chiral $(0, J_2)$ primary gives a generic superfield with (J_1, J_2), $\Delta = \Delta^c + \Delta^a$ and $r = \frac{2}{3}(\Delta^c - \Delta^a)$. By multiplying a *conserved current* superfield $J_{\alpha_1\ldots\alpha_{2J_1},\dot\alpha_1\ldots\dot\alpha_{2J_2}}$ by a chiral scalar superfield one gets a semi–conserved superfield with $\Delta = \Delta^c + 2 + J_1 + J_2$ and $r = \frac{2}{3}(\Delta - 2 - 2J_2)$.

These are the basic rules to construct operators with protected dimensions beside the chiral ones, and they also apply in superconformal field theories of lower or higher dimensions.

Since the anomalous dimensions of the protected operator is fixed in terms of their spin and R–symmetry, it must be given by a rational number. This condition severely restricts the search for the corresponding supergravity states, as it imposes strong constraints on the allowed masses and matter group quantum numbers.

We find in our analysis that the requirement for the anomalous dimensions to be rational implies that one must look for dual states also having rational masses .

The virtue of KK harmonic analysis on a coset space hinges on the possibility of reducing the computation of the mass eigenvalues of the various kinetic differential operators to a completely algebraic problem, while it allows to eliminate completely any explicit dependence on the coordinates of the internal manifold. Harmonics are uniquely identified by G quantum numbers, and they are acted upon by derivatives that are reduced to algebraic operators. Such elegant technique can be quite cumbersome for complicated cosets [6, 15], but it is quite straightforward for the simple T^{11} manifold, where it leads beyond the computation of the scalar laplacian eigenvalues [11], or of specific sectors of the mass spectrum [12].

By diagonalizing different operators for fields of various spin, we have found that all the masses have a fixed dependence on the scalar laplacean eigenvalue

$$H_0(j,l,r) = 6[j(j+1) + l(l+1) - 1/8r^2] \quad (3.11)$$

where (j, l, r) refer to the $SU(2) \times SU(2)$ and to the R–symmetry quantum numbers.

This gives us a new element in the analysis as we will soon see, since besides the $SU(2,2|1)$ quantum numbers, we have also to match those of the matter group.

The full analysis [10] reveals that the supergravity theory has one long graviton multiplet with conformal dimensions

$$\Delta = 1 + \sqrt{H_0(j,l,r) + 4}, \quad (3.12)$$

four long gravitino multiplets with

$$\begin{aligned} \Delta &= -1/2 + \sqrt{H_0(j,l,r\pm 1) + 4}, \\ \Delta &= 5/2 + \sqrt{H_0(j,l,r\pm 1) + 4}, \end{aligned} \quad (3.13)$$

and four long vector multiplets, with

$$\begin{aligned} \Delta = -2 + \sqrt{H_0(j,l,r) + 4}, \\ \Delta = 4 + \sqrt{H_0(j,l,r) + 4}, \\ \Delta = 1 + \sqrt{H_0(j,l,r\pm 2) + 4}. \end{aligned} \quad (3.14)$$

Beside these long ones, there are the shortened supermultiplets.

The above formulae clearly show that the conformal dimensions become rational when the square roots assume rational values

$$H_0 + 4 \in \mathbb{Q}^2. \quad (3.15)$$

This equation is found to admit some special solutions for

$$j = l = |r/2|, \quad (3.16)$$

$$j = l - 1 = |r/2| \text{ or } l = j - 1 = |r/2|. \quad (3.17)$$

Given these strong constraints on the possible $SU(2,2|1)$ quantum numbers as well as on the $SU(2) \times SU(2)$ ones, it becomes an easy task to build the appropriate conformal operators satisfying such constraints and find the relevant bulk supermultiplets.

While referring to [5] for all details, we list some interesting examples of conformal operators.

The chiral operators of the conformal field theory are given by

$$\begin{aligned} S^k &= Tr(AB)^k & (3.18)\\ \Phi^k &= Tr\left[W^2(AB)^k\right] & (3.19)\\ T^k &= Tr\left[W_\alpha(AB)^k\right] & (3.20) \end{aligned}$$

and are shown to correspond to hyper–multiplets containing massive recursions of the dilaton or

the internal metric (3.18 and 3.19) or to tensor multiplets (3.20).

Even more interesting are the towers of operators associated to the semi–conserved currents. Some of them are

$$J^k_{\alpha\dot\alpha} = Tr(W_\alpha e^V \bar W_{\dot\alpha} e^{-V}(AB)^k), \quad (3.21)$$
$$J^k = Tr(Ae^V \bar A e^{-V}(AB)^k), \quad (3.22)$$

which lead to short multiplets whose highest state is a spin 2 and spin 1 field respectively, with masses given by

$$M_{J^k_{\alpha\dot\alpha}} = \sqrt{\frac{3}{2}k\left(\frac{3}{2}k+4\right)}, \quad (3.23)$$
$$\text{and} \quad M_{J^k} = \sqrt{\frac{3}{2}k\left(\frac{3}{2}k+2\right)}. \quad (3.24)$$

These bulk states correspond to massive recursion of the graviton and of the gauge bosons of the matter groups.

It has been explained that under certain conditions the semi–conserved superfields can become conserved, and this is indeed the case. If we set $k = 0$ we retrieve the conserved currents related to the stress–energy tensor and the matter isometries . In fact $M_{J^0_{\alpha\dot\alpha}} = M_{J^0} = 0$ are the massless graviton and gauge bosons of the supergravity theory.

The above analysis can be carried out for M–theory compactifications, where again a full correspondence can be established for the *short* operators on the CFT and the *short* multiplets of the supergravity theory. We must say however that, while in the T^{11} case the superpotential gives us a rule to discard all the sets of operators which are not related to any KK state, for the M–theory KK spectra to agree with the CFT operators one has to uncover some unknown quantum mechanism [6] or the existence of some highly non trivial superpotential [15] that would eliminate the mismatching states.

Up to now we have checked the AdS/CFT correspondence as far as what the conformal field theory imposes on the bulk states, but what can we learn *on the CFT* from the analysis of the supergravity states?

4. Supergravity predictions

There are essentially two aspects of the supergravity theory which can give us new insight on the dual CFT. The first is the presence of long multiplets that nevertheless have rational scaling dimensions, which could provide us with new non–renormalization theorems (at least in the large N, g_sN limit). The other is is the existence of the so–called Betti multiplets, which give rise to additional symmetries in the boundary theory.

Let us now turn to the first aspect.

We have shown that the conformal operators with protected dimension are given by chiral ones or by their products with the conserved currents. The surprising output of the supergravity analysis is that there exist some conformal operators that in spite of not being protected by supersymmetry, still have rational conformal dimension.

Confining ourselves to the T^{11} case, if we take the chiral operator

$$Tr(W^2(AB)^k),$$

we can make it non–chiral by simply inserting into the trace an antichiral combination of the gauge field–strength

$$Tr(W^2 e^V \bar W^2 e^{-V}(AB)^k).$$

This operator then corresponds to a long multiplet in the bulk theory and one should expect its scaling dimension to be generically renormalized to an irrational number. If we search for the corresponding vector multiplet in the supergravity theory, we see that its anomalous dimension is instead rational and *matches exactly the naive sum of the dimensions of the operators inside the trace*. We find this to be the case for all the lowest non–chiral operators of general towers with irrational scaling dimension. For instance, the towers of operators

$$Tr\left[W_\alpha(Ae^V\bar A e^{-V})^n(AB)^k\right] \quad (4.1)$$
$$Tr\left[e^V \bar W_{\dot\alpha} e^{-V}(Ae^V\bar A e^{-V})^n(AB)^k\right] \quad (4.2)$$

have an irrational value of Δ for generic n, but when $n = 1$ we have found that they do have rational anomalous dimension $\Delta = 5/2 + 3/2k$. When $n = 0$ we retrieve the chiral, or semi–conserved operators with protected Δ. This is

a highly non–trivial prediction of the correspondence on the CFT which comes only from the computation of the spectrum on the KK side and we hope it could receive in the future an explanation from the CFT point of view.

If we restrict our attention to the protected operators, we could say that the above peculiar feature arises also in the AdS_4/CFT_3 case. However, we have a true one–to–one map and full agreement on the two sides only for a specific seven–dimensional compactification, that is the Stiefel manifold $SO(5)/SO(3)$ [15] (see the summary table therein).The latter seems to be rather different from the other $\mathcal{N}=2$ compactifications of [6]. Indeed, although the spectra look very similar, it seems that in the examples dealt with in [6], for some of the supergravity states it is not easy to identify the related CFT operator.

5. Betti multiplets

The second AdS prediction on the CFT is the existence of baryon symmetries.

As pointed out by Witten [19], the existence of such baryon symmetries is related to non–trivial Betti numbers of the internal manifold. Moreover, from the supergravity point of view, the non trivial value of such numbers implies the appearance of extra massless multiplets, the Betti multiplets [10]. It is then quite natural to propose a relation between the existence of Betti multiplets and of baryon symmetries.

Let's see how this works.

The non–trivial b_2 and b_3 numbers of the T^{11} manifold imply the existence of closed non–exact 2–form Y_{ab} and 3-form Y_{abc}. These forms must be singlets under the full isometry group, and thus they signal the presence of new additional massless states in the theory than those connected to the $SU(2)\times SU(2)\times U_R(1)$ isometry.

From the KK expansion of the complex rank 2 A_{MN} and real rank 4 A_{MNPQ} tensors of type IIB supergravity we learn that we should find in the spectrum a massless vector (from $A_{\mu abc}$), a massless tensor (from $A_{\mu\nu ab}$) and two massless scalars (from the complex A_{ab}). This implies the existence of the so called Betti vector, tensor and hyper–multiplets, the last two being a peculiar feature of the AdS_5 compactification [10]. The additional massless vector can be seen to be the massless gauge boson of an additional $U_B(1)$ symmetry of the theory.

From the boundary point of view we need now to find an operator counterpart for such vector multiplet and look for an interpretation of the additional symmetry. The task of finding the conformal operator is very easy, once we take into account that it must be a singlet of the full isometry group and must have $\Delta = 3$. The only operator we can write is [5, 20]

$$\begin{aligned}&\mathcal{U} = Tr\left(Ae^V\bar{A}e^{-V}\right) - Tr\left(Be^V\bar{B}e^{-V}\right)\\&(D^2\mathcal{U} = \bar{D}^2\mathcal{U} = 0),\end{aligned} \tag{5.1}$$

which represents the conserved current of a baryon symmetry of the boundary theory under which the A and B field transform with opposite phase. We have shown that the occurrence of such Betti multiplets is indeed due to the existence of non–trivial two and three–cycles on the T^{11} manifold. This implies that, from the stringy point of view, we can wrap the $D3$–branes of type IIB superstring theory around such 3–cycles and the wrapping number coincides with the baryon number of the low–energy CFT [20].

We would like to point out that this feature of some manifolds can be used to check the right dimension of the singleton fields as done in [6]. One can indeed compute the conformal dimension of the CFT operator coupling to the baryon field obtained by a Dp–brane wrapping a non–trivial p–cycle and match it with its mass, which should be proportional to the volume of the same cycle.

6. A puzzle

An interesting case where the baryonic symmetry does not appear to be simply related to the Betti multiplets is that of type IIA compactification on $AdS_4\times\mathbb{P}^3$. This gives a supergravity theory which should be dual to an $\mathcal{N}=6$ CFT in three dimensions. It has been conjectured that the supergravity spectrum should be the same for M–theory on $AdS_4\times S^7/\mathbb{Z}_k$ (for $k>3$) and for the Hopf reduction of $AdS_4\times S^7$ on $AdS_4\times\mathbb{P}^3$ [21]. It can indeed be shown [22]

that the surviving states of the M–theory expansion on $AdS_4 \times S^7/\mathbb{Z}_k$ are the same as those of the $\mathcal{N} = 8$ theory which are neutral under the $U(1)$ along which we Hopf reduce S^7 to $\mathbb{P}^3$. These are exactly the same as those appearing in the harmonic expansion of type IIA theory on $AdS_4 \times \mathbb{P}^3$.

From these facts, one should deduce that the massless vector of the additional $U(1)$ baryon symmetry is simply the KK vector deriving from the reduction of the eleven dimensional metric on the ten dimensional space $AdS_4 \times \mathbb{P}^3$. But here comes the puzzle.

Type IIA theory has a three–form C which should give rise to Betti vector multiplets when expanded on the internal manifold $\mathbb{P}^3$. The complex projective space $\mathbb{P}^3$ has indeed a non–trivial Betti two–form: the complex structure J_{ab}. This implies that the expansion of $C_{\mu ab}(x, y)$ in terms of the harmonics of the internal manifold contains a vector c^0_μ coupled to this form:

$$C_{\mu ab}(x, y) = \sum_I c^I_\mu(x) Y^I_{ab}(y) + c^0_\mu(x) J_{ab}. \qquad (6.1)$$

This again could be interpreted as the massless vector of the baryon symmetry, but we know we have only one such vector.

The solution lies in the fact that this c^0_μ is non–physical. It is actually a pure gauge mode as we will shortly see.

Usually, type IIA supergravity is described by a one–form A, a two–form B a three–form C and a dilaton Φ with field–strengths:

$$F = dA, \qquad (6.2)$$
$$H = dB, \qquad (6.3)$$
$$G = dC + A\,dB. \qquad (6.4)$$

If we define

$$C' \equiv C + AB, \qquad (6.5)$$

then $dC' = dC + A\; dB - dA\; B$ and the four–form definition becomes

$$G = dC' + FB. \qquad (6.6)$$

At this point G is trivially invariant under

$$\delta C' = dK, \text{ and } \begin{cases} \delta A = d\Lambda \\ \delta C' = 0 \end{cases}, \qquad (6.7)$$

while $\delta B = d\Sigma$ requires $\delta C = F\Sigma$. This implies that the physical invariance of $B_{\mu\nu}(x)$, $\delta B_{\mu\nu}(x) = 2\partial_{[\mu}\Sigma_{\nu]}(x)$ requires $C'_{\mu ab}$ to transform according to

$$\delta C'_{\mu ab}(x, y) = F_{ab}\Sigma_\mu. \qquad (6.8)$$

Keeping only linear terms in (6.8), we get

$$\delta C'_{\mu ab}(x) = J_{ab}\Sigma_\mu(x), \qquad (6.9)$$

which tells us, by comparison with (6.1) now applied to C', that the generic mode c^I_μ is invariant $\delta_\Sigma c^I_\mu = 0$, while $\delta_\Sigma c^0_\mu = \Sigma_\mu(x)$ is a pure gauge field.

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